59b) $f(x)=3x^2+2x+1 \quad [-1,2]$

1^{ST}, calculate (it's a NUMBER)

$$\int_{-1}^{2} 3x^2+2x+1\,dx = \left(x^3+x^2+x\right)\Big|_{-1}^{2}$$

$$= (14)-(-1) = 15$$

2^{ND}, divide $\int_{-1}^{2} f(x)dx$ by the width of the interval.
This will $f(c)$, the height of the rectangle
(and the average value over that interval)

$$\frac{15}{2-(-1)} = 5 = f(c)$$

3^{RD} solve $f(x)=f(c)$

$$3x^2+2x+1=5 \Rightarrow 3x^2+2x-4=0$$

$$\Rightarrow x = \frac{-2\pm\sqrt{2^2-4(3)(-4)}}{2(3)}$$

$$x = \frac{-2\pm\sqrt{52}}{6} = \frac{-2\pm 2\sqrt{13}}{6} = -\frac{1}{3}\pm\frac{\sqrt{13}}{3}$$

So $c = -\frac{1}{3}+\frac{\sqrt{13}}{3}$

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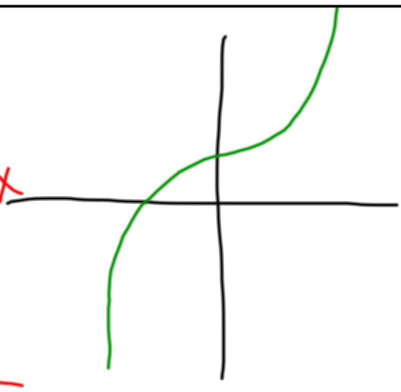
$\sqrt{x^3+2}$ on $[0,3]$

$$\int_0^3 \sqrt{2}\,dx \leq \int_0^3 \sqrt{x^3+2}\,dx \leq \int_0^3 \sqrt{29}\,dx$$



$$3\sqrt{2} \leq \int_0^3 \sqrt{x^3+2}\,dx \leq 3\sqrt{29}$$

$$\int_0^3 \sqrt{2}\,dx = \sqrt{2}\,x \Big|_0^3 = 3\sqrt{2} - 0\sqrt{2} = 3\sqrt{2}$$

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$f(x)$ continuous $\Rightarrow$ $f(x)$ has an antiderivative

"semi-definite" integrals

consider $F(x) = \int_a^x f(t)\,dt$

If $G(t) =$ any antiderivative of $f(t)$

then $\int_a^x f(t)\,dx = G(x) - G(a)$

we get $F(x) = G(x) - G(a)$

$F'(x) = G'(x) - 0 = f(x)$

I.E. $F(x) = \int_a^x f(t)\,dt$

is ALSO an antiderivative of $f(x)$

just like

$f(x) = x^2$

$f(t) = t^2$

$f(b) = b^2$

$f(u) = u^2$

represent the same function

we also have

$\int f(x)\,dx$,

$\int f(t)\,dt$,

$\int f(b)\,db, \ldots$

represent same indef integral

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FTC-2

$f(x)$ continuous $\Rightarrow$ $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

important addition to text

$\Rightarrow$ Note that this is a derivative statement

$\therefore$ the chain rule APPLIES

So FTC-2 extended by bob

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x)) \cdot g'(x)$$

So $\frac{d}{dx}\left[\int_2^{e^{x^2}} \sin(t)\,dt\right] = \sin\left(e^{x^2}\right) \cdot 2xe^{x^2}$

CHAIN RULE

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

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$$\frac{d}{dx}\left[\int_{x^2}^{4} \sin(t)\,dt\right] = -\sin(x^2)\cdot 2x$$

$$= \frac{d}{dx}\left[-\int_{4}^{x^2} \sin(t)\,dt\right] = -\sin(x^2)\cdot 2x$$

$$\frac{d}{dx}\left[\int_{x^2}^{e^{x^2}} \sin(t)\,dt\right] = \left(2xe^{x^2}\cdot \sin(e^{x^2})\right) - (\sin x^2 \cdot 2x)$$

$$\int_{x^2}^{e^{x^2}} \sin(t)\,dt = \int_{x^2}^{0} \sin(t)\,dt + \int_{0}^{e^{x^2}} \sin(t)\,dt$$

6.6/49-55

What does this MEAN?

Derivatives and Antiderivates are inverse processes.

The "slope of tangent line" question and "area under a curve" question are Two sides of the SAME QUESTION.

6.1: two approaches to Area problem
→ antiderivate as area function
→ limit of sums of areas of rectangles
⇒ definite integral

6.6/65

$$\lim_{n \to +\infty} \sum_{k=1}^{n} \underbrace{\frac{\pi}{4n}}_{\text{width of rectangle}} \underbrace{\sec^2\left(\frac{\pi k}{4n}\right)}_{\text{function = height of rectangle}} ; \left[0, \frac{\pi}{4}\right]$$

$$= \int_0^{\frac{\pi}{4}} \sec^2(x)\, dx$$